

## VINCULUM

The secondary journal of The Mathematical Association of Victoria



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## INSIDE

- When will we ever use this?
- Calculator games
- A Pascal pyramid?
- Rectangles in rectangles

## **INSIDE THIS ISSUE**

VOLUME 59, NUMBER 1. 2022

#### COVER

Pink Sorrel - Botanical Geometric Study

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5
D

FROM THE EDITOR

Roger Walter



A RECTANGLE IN A RECTANGLE Roger Walter and Karim Noura

7

CHESS PROBLEM Terence Mills



WHEN WILL I EVER USE THIS? Justine Healey



CALCULATOR GAMES Paul Brown



PASSIONLESS MOMENTS: QUARTIC FUNCTIONS AND THE GOLDEN RATIO Bruce Ruthven



SNIPPET: TWO ANSWERS ENIGMA Roger Walter



THE NEW FOUNDATION MATHEMATICS STUDY DESIGN: IT'S GOOD NEWS FOR STUDENTS Justine Sakurai, Carly Sawatzki and David Tout

PASSIONLESS MOMENTS: SOLUTIONS Bruce Ruthven

CHESS: SOLUTION Terence Mills

21

THE PASCAL-LIKE TRIANGLE: SOLUTIONS Roger Walter



INSIDE THE BACK COVER

Roger Walter

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## THE NEW FOUNDATION MATHEMATICS STUDY DESIGN: IT'S GOOD NEWS FOR STUDENTS

## Justine Sakurai, Carly Sawatzki and Dave Tout

Aisha's learning experiences in mathematics have tended to be procedural and abstract. However, now she has a casual job at Target, she has noticed the implicit numeracy demands that come with this newfound independence. Maybe there is a reason to learn mathematics after all? For example, Target offers AfterPay, and this is something customers ask about. *Is it really interest free? She is also having to keep track* of her payslips and bank balance, and monitor her income and spending. Aisha's parents are encouraging her to save one third of what she earns (with practice, she is becoming better at estimating this amount). She doesn't earn enough to worry about taxation, but has been interested to find out about superannuation, which she now knows is calculated as a proportion of her earnings. And speaking of investing, some of the kids at school are talking about shares and cryptocurrency for getting rich quick. Now Aisha is in the process of selecting her VCE studies, she thinks it would be cool to learn about this stuff at school.

The new VCE Foundation Mathematics Study Design is due for implementation in schools in 2023. For the first time in Victoria, foundation mathematics students will have the opportunity to continue their studies from units 1 & 2 into units 3 & 4. This two-year course will provide a clear pathway for those wanting to continue studying mathematics to the end of their school journey, giving them stronger mathematics knowledge and skills for life beyond school. Research indicates that completing mathematics at school creates positive lifetime outcomes such as increased employability and higher wages (Parsons & Bynner, 2005; Shergold, 2020). In this article we explain why the latest iteration of foundation mathematics is good news for students.

## **STUDY DESIGN AIMS**

The study design aims to keep kids connected with mathematics and build their numeracy for life beyond school. It is a sad reality that many students come to feel disconnected from the discipline of mathematics over the course of their schooling. Students often report that the mathematics they encounter at school seems irrelevant. They express their frustration via the question, 'When am I going to use this?' This is something that this new VCE foundation mathematics study design seeks to fix, based on the knowledge that it's never too late to learn.

Research shows that the knowledge and skills now needed to succeed in work, life and the community as an active and informed citizen are dynamically changing. This is often driven by technological advances and an ever-increasing use of quantitative information and data, but also relates to the transforming nature of the workforce associated with *Industry 4.0* and the *Gig economy*, with increasing demands for science, technology, engineering and mathematics (STEM) skills (e.g. see AAMT and AiGroup 2014; Foundation for Young Australians 2017; Gravemeijer et al 2017; Griffin et al. 2012; Hoyles et al. 2010; NCTM 2017; P21 2016; Pellegrino et al. 2012).

However, as Gravemeijer and colleagues (2017, p.109) summarised:

Despite calls from many reports, research, and mathematics organizations to contextualize mathematics, many schools continue to present mathematics as abstract, decontextualized problems.

Students at school need opportunities to develop the ability to make considered, mathematically informed decisions, whether they be related to personal financial matters, planning short commutes and longer travels, interpreting big data such as with the current COVID-19 epidemic, following instructions about a health or medical matter, or understanding local, national or global environmental issues arising from climate change. This requires students to be actively engaged in solving real world problems where mathematics may not be readily identifiable but is certainly embedded in a range of different contexts. The new VCE foundation mathematics study design is designed to help students see the purpose and use of mathematics to their current and future lives, and we believe that this approach can better attract, engage and retain more students to continue to study mathematics to the end of Year 12.

People crave knowledge and skills that they perceive will be useful. The VCE foundation mathematics study design specifies that 'the selected content for each unit should be developed using contexts present in students' other studies, work and personal or other familiar situations, and in national and international contexts, events and developments' (p. 71). The intention is to offer students the best preparation to apply mathematics beyond the classroom – right now, and as young adults interacting in an increasingly complex world.

Teachers are trusted to be experts in their school and students, and to ensure that explicit teaching of mathematical concepts is balanced with opportunities to learn through purposefully selected real-world contexts that will excite young people while expanding their horizons. This autonomy acknowledges that while learning outcomes are defined for all, the situatedness of this learning will vary from school to school. The issues, trends, problems, investigations, illustrations and examples that are most appropriate and engaging to students will be best chosen in consultation with colleagues and students, with a focus on learning mathematics so as to better understand and interact in the world.

A good example is the focus on financial and consumer mathematics within and across units and areas of study. Modelling tasks, problem-solving tasks and mathematical investigations are specified for the purpose of teaching, learning and assessment. One approach would be to

## THE NEW FOUNDATION MATHEMATICS STUDY DESIGN (CONT.)

contextualise your school's program by engaging students in investigating economic and financial issues over the course of the year. For example, when teaching data analysis, probability and statistics, you might plan to explore and analyse workforce data over the course of the pandemic. What do students notice and wonder about in terms of the impact on women (termed the pink recession) and casual workers? When teaching algebra, number and structure, you might revisit and expand upon this learning, with a greater focus on modelling the effect of career interruption on lifetime earnings (termed the gender pay and superannuation gap).

Of course, you may feel that such contexts are outside your comfort zone - that is OK. Meetings that bring together colleagues who teach mathematics *and* commerce can be mutually informative, and bring new perspectives and expertise to each others' lesson planning. Work together to look beyond the textbook – mathematics is all around you!

## THIS STUDY DESIGN ENCOURAGES THE USE OF CURRENT TECHNOLOGIES

Modern life is awash with digital technologies. The Australian Media and Communications Authority recently reported that 97% of young people aged 18-34 use their mobile phones to go online (AMCA, 2021). This generation are digital natives, having grown up using a range of sophisticated devices they can take with them on-the-go (i.e., smart phones and tablets). They are entering or immersed in workplaces that are embracing technology as the norm.

The new VCE foundation mathematics study design acknowledges this new reality with no one type of technology specified. The curriculum notes the importance of using modern digital technologies with the study dot point below.

awareness and knowledge of contemporary technological and online and digital media, including software and applications based on computers, tablets, calculators and hand-held devices (VCAA, 2021, p. 14)

Educational programs can be designed around the range of technologies that exist in the lives of students, and what is most appropriate to your school setting, with consideration for what technologies might be accessed through industry or work contexts.

Given the power of technology available in schools today, the importance of learning to think with technology is accepted as best practice in the mathematics classroom. Computational thinking (Weintrop et al., 2016, p.134) can be broken into four main categories.

- Data practices
- Modelling and simulation practices
- Computational problem solving practices
- Systems thinking practices

Considering each of these practices separately demonstrates how we might design activities and tasks for the classroom.

**Data practices** encompass ideas of statistics and probability. In the retail industry data is stored using spreadsheets to capture information around stock levels and staff hours and schedules. Bringing this alive in the classroom allows for meaningful mathematics learning to occur (Geiger et al., 2015; Gravemeijer et al., 2017). Tasks can be designed around learning and using spreadsheets such as Excel or GoogleSheets.

**Modelling and simulation practices** can be employed in the classroom to mimic real world problems (Arseven, 2015). The importance of using modelling as part of problem solving is vital in developing students who can participate effectively in modern Australia (Doerr et al., 2017). This has been particularly evident with the COVID-19 pandemic, and the complex health modelling that was consistently presented to the public to explain the restrictions on society. In the classroom, teachers may want to access one of the many programs such as Geogebra that include pre-designed models with variables that can be manipulated in the process of discovery. Alternatively, teachers may undertake a real world problem and use a graphical application such as Desmos or Google charts to present their model.

**Computational problem-solving practices** involve the understanding that computers have the power to present solutions to problems much quicker than the human mind. Selecting the correct technology for the task is one aspect of this subcategory. Teachers may ask their students to choose and justify the reasoning for their selection. Some digital tools require calibration or setting up of parameters for use. In the health industry, nurses use electronic intravenous drip machines to introduce liquids to patients. Whilst it is not likely that teachers can access this specific technology in the classroom, the ideas and skills of calibration have transferability. Think of the resources at your disposal, large photocopiers need calibration and so do coffee machines. Students should understand how to use the technology to express a mathematical problem and this is a skill that can be explicitly taught (Weintrop et al., 2016).

**Systems thinking practices** include the ability to see the big picture. Sure, your students can use a location app on their smartphone to follow step-by-step directions to get from point A to point B. But do your students understand the representations and conventions on the map? If the map asks them to follow a road that does not exist, can they examine the technology and re-route? Location apps are a feature of modern lives, and it is now uncommon to find a hardcopy map in everyday use.

#### 14 VINCULUM: VOLUME 59, NUMBER 1. 2022 © The Mathematical Association of Victoria

## CALCULATOR GAMES (CONT.)

## **CONCLUDING REMARKS**

Students should complete their school studies with the essential mathematical knowledge and skills to take them through life, whatever the future may bring. Through real world contexts, a range of new technologies and research-informed pedagogies, you can make VCE foundation mathematics what your students need it to be to enter life beyond school with confidence.

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 $(n^2 + kn + 1)(n^2 + kn + 1) = n^4 + 2kn^3 + \dots$ 

 $n(n + 1)(n + 2)(n + 3) + 1 = n^4 + 6n^3 + \dots$ 

This gives 2k = 6, k = 3, but we must still check our result by expanding  $(n^2 + 3n + 1)^2$ .

#### **STRATEGY 3**

Since this is a calculator activity, you could always use a CAS calculator to factorise  $n^4 + 6n^3 + 11n^2 + 6n + 1$ .

## **GAMES 6 AND 7: CONTINUED FRACTIONS**

The more steps you do, the closer the answer comes to  $\phi + 1$ .

When you solve the equation, you end up with  $Q^2 - Q - 1 = 0$ , which gives

$$Q = \frac{1+\sqrt{5}}{2}$$
 or  $Q = \frac{1-\sqrt{5}}{2}$ 

The first solution is  $\phi + 1$  which also happens to equal to  $1/\phi$ . The second solution is  $-\phi$ .

When you use  $Q = 1 - \frac{1}{Q}$  you find it is always a repeating sequence of three numbers. You can show that starting with any number *n* gives  $1 - \frac{1}{n} = \frac{n-1}{n}, -\frac{1}{n-1}, n$ , repeating indefinitely. For example, if n = 2, we get 0.5, -1, 2, 0.5, -1, 2, ...

$$Q = 1 - \frac{1}{Q} \Longrightarrow Q^2 - Q + 1 = 0.$$

This equation has no real solutions ( $\Delta = -3$ ), so we don't get anywhere with our continued fractions.

### GAME 8: ABSURD

These numbers are the solutions of the same quadratic equation as game 6,  $Q^2 - Q - 1 = 0$ .

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